EFFECT OF SHIELDS ON RADIATIVE HEAT TRANSFER WITHIN THE PENETRATION ZONE

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A procedure is proposed for calculating the effect of thin "floating" shields and of cooled cylindrical shields on radiative heat transfer within the penetration zone.

The effect of shields on radiative heat transfer within the penetration zone (e.g., within the reactor radiation zone) is a problem which requires a separate study, because part of the penetrating radiant energy is dissipated in a shield in the form of heat and thus affects the heat transfer between surfaces insulated from one another. When a shield is exposed to a nonuniform heat load, then evidently its temperature field becomes also nonuniform and internal heat is generated in an amount which depends on the thermal conductivity of the shield material. It is important, in this case, to correctly estimate the effect of not only the shield as a whole but also of its individual segments on the heat transfer.

The equations derived in [1], where the thermal effect of penetrating radiation on a shield is represented by an additional flux impinging on the shield and by a mean shield temperature, fully reflect the effect of the shield set as well as of each individual shield segment on the thermal conditions, but do not define the role of individual shield segments in the heat transfer.

A convenient method of estimating the effectiveness of individual shield segments would probably be by comparing the temperature field of a given shield with the surface temperature of neighboring objects.

Thin "Floating" Shields. In determining the effectiveness of individual segments of a "floating" shield (i.e., a shield which participates in heat transfer with other objects by radiation only), one must, for certain reasons, disregard the heat transfer between a given segment and the other segments of the same shield. Namely, in the total heat transfer between different shield segments by radiation and conduction the role of each is blurred because, although part of the heat which the segment most intensively heated by penetrating radiation receives is transmitted further to other segments, this part is still involved in radiative heat transfer between those other segments and the surfaces of neighboring objects and, therefore, in the end still bears on the effectiveness of the thermal insulation. As a result of heat transfer between different shield segments, the temperature becomes rather equalized over the entire shield (the temperature of some segments rises, causing the temperature of other segments to drop) and it can possibly remain within the allowable limit throughout the shield, but in reality some individual shield segments will reduce the effectiveness of thermal insulation.

In Fig. 1 are shown the components of a closed system where heat is transferred radiatively between surfaces A and B, with a thin "floating" shield Sh between them which splits system AB into two closed systems. When one among many shields is considered, then obviously the surfaces of adjacent shields must be taken as the surfaces A and B. We will assume that $T_A > T_B$, that all surfaces are gray, and that the directional radiation coefficients are each constant over the respective surface. It may also be assumed, if the shield is thin-walled and of uniform thickness, that its lateral surfaces have equal areas. In this case the shield is most conveniently replaced by the surface of its median layer.

The steady-state heat balance in a shield element conforming to these concepts can be written down as

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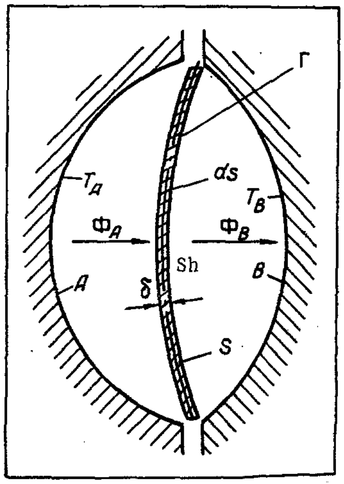

Fig. 1. Schematic diagram of a "floating" shield.

$$d\Phi_A + d\Gamma - d\Phi_B = 0, \tag{1}$$

where

$$d\Phi_A = \sigma\varepsilon_{\mathrm{ref},1}\varphi_{\mathrm{ShA}}(T_A^4 - T_N^4)\,dS; \quad d\Gamma = \rho\delta\gamma_N dS;$$
$$d\Phi_B = \sigma\varepsilon_{\mathrm{ref},2}\varphi_{\mathrm{ShB}}(T_N^4 - T_B^4)\,dS.$$

We disregard the temperature difference between opposite points on the two shield surfaces. After the necessary substitution and transformation, we have

$$T_N^4 = \frac{\varepsilon_{\mathrm{ref},1}\varphi_{\mathrm{ShA}}T_A^4 + \varepsilon_{\mathrm{ref},2}\varphi_{\mathrm{ShB}}T_B^4 + \rho\delta\gamma_N/\sigma}{{}_{\mathrm{ref},1}\varphi_{\mathrm{ShB}} + {}_{\mathrm{ref},2}\varphi_{\mathrm{ShB}}}. \tag{2}$$

The mean shield temperature can be calculated from the condition:

$$\Phi_A = \sigma\varepsilon_{\mathrm{ref},1}\varphi_{\mathrm{ShA}}S(T_A^4 - T_{\mathrm{Sh}}^4) = \int_S d\Phi_A.$$

After the necessary substitution and transformation, letting $\Gamma = \rho\delta\int_S \gamma_N dS$, we have

$$T_{\mathrm{Sh}} = \left(\frac{1}{S}\int_S T_N^4 dS\right)^{\frac{1}{4}} = \left[\frac{\varepsilon_{\mathrm{ref},1}\varphi_{\mathrm{ShA}}T_A^4 + \varepsilon_{\mathrm{ref},2}\varphi_{\mathrm{ShB}}T_B^4 + \Gamma/(\sigma S)}{\varepsilon_{\mathrm{ref},1}\varphi_{\mathrm{ShA}} + \varepsilon_{\mathrm{ref},2}\varphi_{\mathrm{ShB}}}\right]^{\frac{1}{4}}. \tag{3}$$

The same result is obtained from the condition that $\Phi_B = \int_S d\Phi_B$.

It is easy to see that formula (3) represents a special case of the solution to the equation derived in [1].

a) When one considers protecting the relatively cold surface, i.e., surface B in Fig. 1 against increments of heat, one must take into account that heating the shield will reduce the effectiveness of shielding the insulated surface. A case in point is the thermal insulation of low-temperature apparatus installed within the radiation penetration zone. It is convenient here to compare the temperature field of the shield with the temperature of the relatively hot surface, i.e., with the temperature T_A. If one finds that the temperature of any shield segment is above T_A, this segment reduces the thermal insulation and must be removed. This requirement can be expressed as $T_N \leq T_A$. After the necessary substitution and transformation, we have

$$\rho\delta\gamma_N \leqslant \sigma\varepsilon_{\mathrm{ref},2}\varphi_{\mathrm{ShB}}(T_A^4 - T_B^4). \tag{4}$$

When the maximum thermal flux per unit mass supplied to the shield by penetrating radiation is known, then the critical thickness for a shield of a given material can be calculated by the formula

$$\delta_{\mathrm{cr}} = \frac{\sigma\varepsilon_{\mathrm{ref},2}\varphi_{\mathrm{ShB}}(T_A^4 - T_B^4)}{\rho\gamma_{\mathrm{max}}}, \tag{5}$$

while the critical flux supplied per unit shield mass of given thickness and material is

$$\gamma_{\mathrm{cr}} = \frac{\sigma\varepsilon_{\mathrm{ref},2}\varphi_{\mathrm{ShB}}(T_A^4 - T_B^4)}{\rho\delta}. \tag{6}$$

When there are n shields in a set, the formula for the resulting thermal radiation flux at the insulated surface B is in this case [1]:

$$\Phi_B = \sigma\varepsilon_{AB}H_{AB}(T_A^4 - T_B^4) + \sum_{k=1}^{n}\left(\Gamma_k\frac{\varepsilon_{AB}H_{AB}}{\varepsilon_{Ak}H_{Ak}}\right) = \frac{\sigma(T_A^4 - T_B^4)}{\sum_{i=1}^{n+1}\frac{1}{\varepsilon_{\mathrm{ref},i}H_i}} + \sum_{k=1}^{n}\left(\Gamma_k\frac{\sum_{i=1}^{k}\frac{1}{\varepsilon_{\mathrm{ref},i}H_i}}{\sum_{i=1}^{n+1}\frac{1}{\varepsilon_{\mathrm{ret},i}H_i}}\right). \tag{7}$$

The last formula indicates that more shields in the penetration zone, with all other conditions unchanged, will increase the shielding effectiveness, i.e., will reduce the magnitude of Φ_B, up to a certain number of shields above which the shielding effectiveness will again decrease. As is well known, increasing the number of "floating" shields outside the irradiation zone will always improve the shielding effectiveness [1].

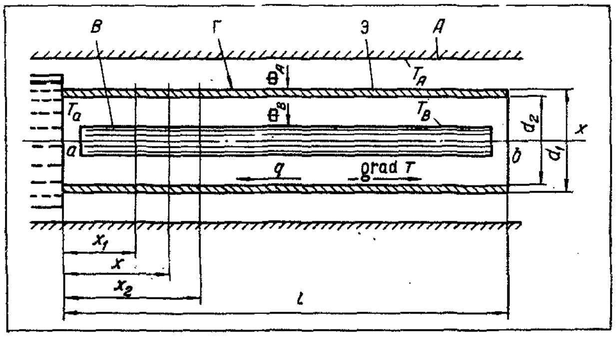

Fig. 2. Schematic diagram of a cylindrical shield cooled at the end surface.

In order to determine the optimum number of "floating" shields inside the reactor radiation zone, it is evidently necessary to find the minimum of function $\Phi_B(n)$ from formula (7).

b) When one considers protecting the relatively hot surface, i.e., surface A in Fig. 1 against heat loss, one must take into account that heating the shield will improve its effectiveness until $T_{Sh} = T_A$ and full shielding is attained. Further heating of the shield will again reduce its effectiveness.

In order to determine the optimum thermal flux supplied to the shield by penetrating radiation, we substitute (3) for T_{Sh} in the condition $T_{Sh} = T_A$. A few minor transformations yield

$$\Gamma_{opt} = \sigma\varepsilon_{ref,2}\varphi_{ShB}S(T_A^4 - T_B^4). \tag{8}$$

In this case the resulting thermal radiation flux between shield surface and surface A is equal to zero, and all extraneous heat Γ_{opt} is transmitted to surface B.

With the material and the irradiation spectrum of the shield known, the optimum shield thickness can be calculated by the formula:

$$\delta_{opt} = \Gamma_{opt} \Big/ \Big(\rho \int_S \gamma_N dS\Big), \tag{9}$$

while with the shield material and thickness known, the optimum mean thermal flux per unit mass supplied to the shield is

$$\gamma_{opt} = \Gamma_{opt}/(\rho\delta S). \tag{10}$$

When there are n shields in a set, the formula for the resulting thermal radiation flux to the insulated surface A is in this case [1]

$$\Phi_A = \sigma\varepsilon_{AB}H_{AB}(T_A^4 - T_B^4) - \sum_{k=1}^{n}\left(\Gamma_k \frac{\varepsilon_{AB}H_{AB}}{\varepsilon_{kB}H_{kB}}\right) = \frac{\sigma(T_A^4 - T_B^4)}{\sum\limits_{i=1}^{n+1}\frac{1}{\varepsilon_{ref,i}H_i}} - \sum_{k=1}^{n}\left(\Gamma_k \frac{\sum\limits_{i=k+1}^{n+1}\frac{1}{\varepsilon_{ref,i}H_i}}{\sum\limits_{i=1}^{n+1}\frac{1}{\varepsilon_{ref,i}H_i}}\right). \tag{11}$$

The optimum number of shields, with all other conditions unchanged, can evidently be determined according to formula (11) for the condition $\Phi_A = 0$.

Cylindrical Shield Cooled at the End Surface. In low-temperature apparatus, the heatup of the shields by penetrating radiation reduces their effectiveness. In such cases it is often possible to improve the shielding effectiveness by cooling the shields. The extra thermal flux supplied to the shield [1] is the algebraic difference between the thermal flux supplied by penetrating radiation and the thermal flux removed by cooling.

An end-surface cooling scheme for a cylindrical shield is shown in Fig. 2. The vacuum jacket A, the shield Sh, and the insulated object B constitute coaxial cylinders. One end surface of the shield is cooled and maintained at a constant temperature. If the cooling of the shield is considered as a one-dimensional heat process, then the mean temperature of any shield section will be

$$T_x = T_a + \frac{1}{\lambda}\int_0^x q_{x_1}dx_1 = T_a + \frac{1}{\lambda F}\int_0^x (\Phi_{A,x_1b} - \Phi_{B,x_1b} + \Gamma_{x_1b})\,dx_1.$$

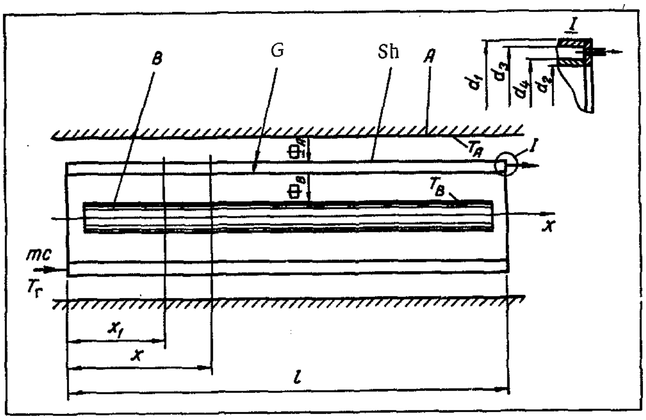

Fig. 3. Schematic diagram of a cylindrical shield cooled by a through-feed of gas.

Here Φ_{A,x_1b}, Φ_{B,x_1b}, and Γ_{x_1b} are thermal fluxes per shield segment of length $l-x_1$, i.e., per segment x_1b. These thermal fluxes are functions of x, and they can be calculated by the following formulas:

$$\Phi_{A,x_1b}=\int_{x_1}^{l} d\Phi_A=\sigma\varepsilon_{\mathrm{ref},1}\varphi_{\mathrm{ShB}}\int_{x_1}^{l}(T_A^4-T_{x_2}^4)\,dS_1=\sigma\varepsilon_{\mathrm{ref},1}\,\pi d_1\int_{x_1}^{l}(T_A^4-T_{x_2}^4)\,dx_2,$$

$$\Phi_{B,x_1b}=\int_{x_1}^{l} d\Phi_B=\sigma\varepsilon_{\mathrm{ref},2}\varphi_{\mathrm{ShB}}\int_{x_1}^{l}(T_{x_2}^4-T_B^4)\,dS_2=\sigma\varepsilon_{\mathrm{ref},2}\varphi_{\mathrm{ShB}}\pi d_2\int_{x_1}^{l}(T_{x_2}^4-T_B^4)\,dx_2,$$

$$\Gamma_{x_1b}=\int_{x_1}^{l} d\Gamma=\rho F\int_{x_1}^{l}\gamma_{x_2}dx_2.$$

After the necessary substitution and transformation, we have

$$T_x=T_a+A_1\left(lx-\frac{x^2}{2}\right)+A_2\int_0^x\left(\int_{x_1}^{l}\gamma_{x_2}dx_2\right)dx_1-A_3\int_0^x\left(\int_{x_1}^{l}T_{x_2}^4dx_2\right)dx_1, \qquad (12)$$

where

$$A_1=\frac{\sigma\pi}{\lambda F}(\varepsilon_{\mathrm{ref},1}d_1T_A^4+\varepsilon_{\mathrm{ref},2}\varphi_{\mathrm{ShB}}d_2T_B^4);\quad A_2=\frac{\rho}{\lambda};$$

$$A_3=\frac{\sigma\pi}{\lambda F}(\varepsilon_{\mathrm{ref},1}d_1+\varepsilon_{\mathrm{ref},2}\varphi_{\mathrm{ShB}}d_2).$$

On the right-hand side of this equation there appears the unknown variable, which is found by methods of approximate integration. The critical shield length is found from the condition $T_x = T_A$.

The temperature of the uncooled shield end is

$$T_b=T_a+A_1\frac{l^2}{2}+A_2\int_0^l\left(\int_{x_1}^{l}\gamma_{x_2}dx_2\right)dx_1-A_3\int_0^l\left(\int_{x_1}^{l}T_{x_2}^4dx_2\right)dx_1.$$

The mean shield temperature can be determined from the condition

$$\Phi_A=\sigma\varepsilon_{\mathrm{ref},1}\pi d_1l(T_A^4-T_{\mathrm{Sh}}^4)=\int_0^l d\Phi_A=\sigma\varepsilon_{\mathrm{ref},1}\pi d_1\int_0^l(T_A^4-T_x^4)\,dx$$

or

$$\Phi_B=\sigma\varepsilon_{\mathrm{ref},2}\varphi_{\mathrm{ShB}}\pi d_2l(T_{\mathrm{Sh}}^4-T_B^4)=\int_0^l d\Phi_B=\sigma\varepsilon_{\mathrm{ref},2}\varphi_{\mathrm{ShB}}\pi d_2\int_0^l(T_x^4-T_B^4)\,dx.$$

After the necesary transformation, we have

$$T_{Sh} = \left(\frac{1}{l} \int_0^l T_x^4 dx \right)^{\frac{1}{4}}. \tag{13}$$

Cylindrical Shield Cooled by a Through-Feed of Gas (Fig. 3). The insulated object B, the shield Sh, and the vacuum jacket A constitute coaxial cylinders. The shield is formed by two coaxial tubes. Cooling gas is fed through the annular space between both tubes.

With heat transmission along the shield disregarded, the steady-state temperature of any shield section can be determined from the condition of heat balance:

$$T_x = T_G + \Delta T_{g,x} + \Delta T_{Sh,x} = T_G + \frac{\Phi_{A,x} - \Phi_{B,x} + \Gamma_x}{mc} + \frac{d\Phi_{A,x} - d\Phi_{B,x} + d\Gamma_x}{a dS_G}.$$

Here $\Phi_{A,x}$, $\Phi_{B,x}$, and Γ_x are thermal fluxes per shield segment of length x. These fluxes are functions of x and they can be calculated by the following formulas:

$$\Phi_{A,x} = \int_0^x d\Phi_A = \int_0^x \sigma\varepsilon_{ref,1}(T_A^4 - T_{x_1}^4)\, dS_1 = \sigma\varepsilon_{ref,1}\pi d_1 \int_0^x (T_A^4 - T_{x_1}^4)\, dx_1,$$

$$\Phi_{B,x} = \int_0^x d\Phi_B = \int_0^x \sigma\varepsilon_{ref,2}\varphi_{ShB}(T_{x_1}^4 - T_B^4)\, dS_2 = \sigma\varepsilon_{ref,2}\varphi_{ShB}\pi d_2 \int_0^x (T_{x_1}^4 - T_B^4)\, dx_1,$$

$$\Gamma_x = \int_0^x d\Gamma = \rho F \int_0^x \gamma_{x_1} dx_1, \text{ and } dS_G = \pi(d_3 + d_4)\, dx_1.$$

After the necessary substitution and transformation, we have

$$T_x = A_1 + A_2 x + A_3\gamma_x + A_4 \int_0^x \gamma_{x_1} dx_1 - A_5 T_x^4 - A_6 \int_0^x T_{x_1}^4 dx_1, \tag{14}$$

where

$$A_1 = T_r + \frac{\sigma(\varepsilon_{ref,1} d_1 T_A^4 + \varepsilon_{ref,2}\varphi_{ShB} d_2 T_B^4)}{a(d_3 + d_4)};$$

$$A_2 = \frac{\sigma\pi(\varepsilon_{ref,1} d_1 T_A^4 + \varepsilon_{ref,2}\varphi_{ShB} d_2 T_B^4)}{mc};$$

$$A_3 = \frac{\rho F}{a\pi(d_3 + d_4)}; \quad A_4 = \frac{\rho F}{mc};$$

$$A_5 = \frac{\sigma(\varepsilon_{ref,1} d_1 + \varepsilon_{ref,2}\varphi_{ShB} d_2)}{a(d_3 + d_4)} \text{ and } A_6 = \frac{\sigma\pi(\varepsilon_{ref,1} d_1 + \varepsilon_{ref,2}\varphi_{ShB} d_2)}{mc}.$$

This equation, like Eq. (12), is solved by methods of approximate integration, and the critical shield length is found from the condition $T_x = T_A$.

The mean temperature of a shield of length l is determined with the aid of formula (13).

NOTATION

- T_A, T_B are the mean temperatures (°K) of the relatively hot and of the relatively cold surface, respectively, both surfaces insulated from one another;
- T_{Sh} is the temperature of the shield;
- T_N is the mean temperature of an element of a "floating" shield defined by point N(x, y, z);
- T_x is the mean temperature of an arbitrary section of a cooled cylindrical shield defined by coordinate x;
- T_a is the mean temperature of the cooled end surface of a cylindrical shield;
- T_b is the mean temperature of the uncooled end surface of a cylindrical shield;
- T_G is the temperature of the cooling gas;
- $\Delta T_{g,x}$ is the temperature rise of the gas along a shield segment of length x;
- $\Delta T_{Sh,x}$ is the mean temperature drop between the shield and the cooling gas, at a shield section defined by coordinate x;

Φ_A	is the resultant thermal radiation flux from the relatively hot surface A to the shield;
Φ_B	is the resultant thermal radiation flux to the relatively cold surface B;
γ	is the thermal flux per unit mass supplied to the shield by penetrating radiation;
Γ	is the thermal flux supplied to the shield by the penetrating radiation;
σ	is the Stefan–Boltzmann constant;
$\varepsilon_{ref,1}$	is the referred emissivity of the closed radiative heat transfer system consisting of the shield and surface A;
$\varepsilon_{ref,2}$	is the referred emissivity of the closed radiative heat transfer system consisting of the shield and surface B;
φ_{ShA}	is the mean directional radiation coefficient from the shield to surface A;
φ_{ShB}	is the mean directional radiation coefficient from the shield to surface B;
ε_{AB}	is the referred emissivity of the set of closed systems formed by the shields between surfaces A and B;
H_{AB}	is the interradiation surface of the closed system AB without shields between A and B;
S	is the one lateral surface area of a "floating" shield;
δ	is the thickness of a "floating" shield;
q	is the thermal flux per unit cross section area;
F	is the cross section area of a shield;
λ	is the thermal conductivity of the shield material;
ρ	is the density of the shield material;
S_1, S_2	are the areas of outside (facing A) and of inside (facing B) surface of a cylindrical shield, respectively;
d_1, d_2	are the diameters of those respective surfaces;
S_G	is the total area of the cylindrical surface, of outer and inner shield tube cooled by a through-feed of gas;
l	is the length of the shield;
m	is the mass flow rate of the gas stream;
c	is the mean specific heat of the cooling gas;
α	is the mean coefficient of the heat transfer from the shield walls to the gas;
A, B	are the two insulated surfaces;
Sh	is the shield;
x	is the axial coordinate.

LITERATURE CITED

1. L. A. Vadachkoriya, Inzh.-Fiz. Zh., 18, No. 1, 21 (1970).